Société de Calcul Mathématique SA
Algorithmes et Optimisation

Fédération Française des Jeux Mathématiques

Mathematical Competitive Game 2012-2013

Fighting Forest Fires in Siberia

Fédération Française des Jeux Mathématiques
(French Federation of Mathematical Games)

and

Société de Calcul Mathématique SA

together with

La Brigade des Sapeurs Pompiers de Paris
(The Paris Firemen Brigade)

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I. Presentation of the Games

The "Mathematical Games", jointly organized by FFJM and SCM, have existed for four years ; the previous ones were:

– In 2008-2009, conception of a bus transportation network in a city, in partnership with Veolia Transport ;

– In 2009-2010, conception of an electricity distribution network, in partnership with RTE (Réseau de Transport de l'Electricité) ;

– In 2011-2012, search for the best itinerary by a car, in partnership with the newspaper Auto Plus.

They deal with the resolution of a "real life" problem, that is a problem which concerns everybody, but simplified in the mathematical contents. Still, this resolution typically requires several months of work.

Candidates may compete individually of as groups, for instance high school classes, or college students, or university students, preparing a "memoir" for the end of their studies.

Two categories of prizes are given:

Individual prizes:
For the winner: 500 Euros
For the second: 200 Euros
For the next three: 100 Euros each.

Prizes for groups:
For the winner: 500 Euros
For the second: 200 Euros
For the next three: 100 Euros each.

The total amount of prizes is therefore 2 000 Euros.

The best solutions are published on the web site of FFJM, on the web site of SCM, and on the web site of our partners.

The official announcement of the results and the ceremony for prizes occur during the "Salon de la Culture et des Jeux Mathématiques" (Fair for Mathematical Culture and Games), which is held in Paris, each year, during the month of May.

The winners, previous years, gained considerable notoriety, both in the press and television in their respective countries.

II. The 2012-2013 Prize

A. General presentation of the subject

We try here to help a Government in its decisions, regarding the repartition of the capacities, in order to fight against fires. The difficulty resides in the fact that, as this is always the case in real life situations, necessary data are not correctly known. At best, we have some rough information.

For the previous games, the territory was given under a simplified form (a square). Here, we have a real territory, which is both very large and rather complex. The problem is a true worry for all governments, every year, in all countries.

The problem is given under its most direct presentation, the most common one: we have at our disposal global capacities, of fixed type, in order to fight the fires. Our problem is to use them at best. In usual terms, we will say that our concern is to obtain that fires cause as little damage as possible. We do not try to prevent the fires (this would be impossible in practice), but only to limit their extension. In mathematical terms, we try to minimize the expectation of the cost of the damages, using the capacities at best.

B. The concrete situation

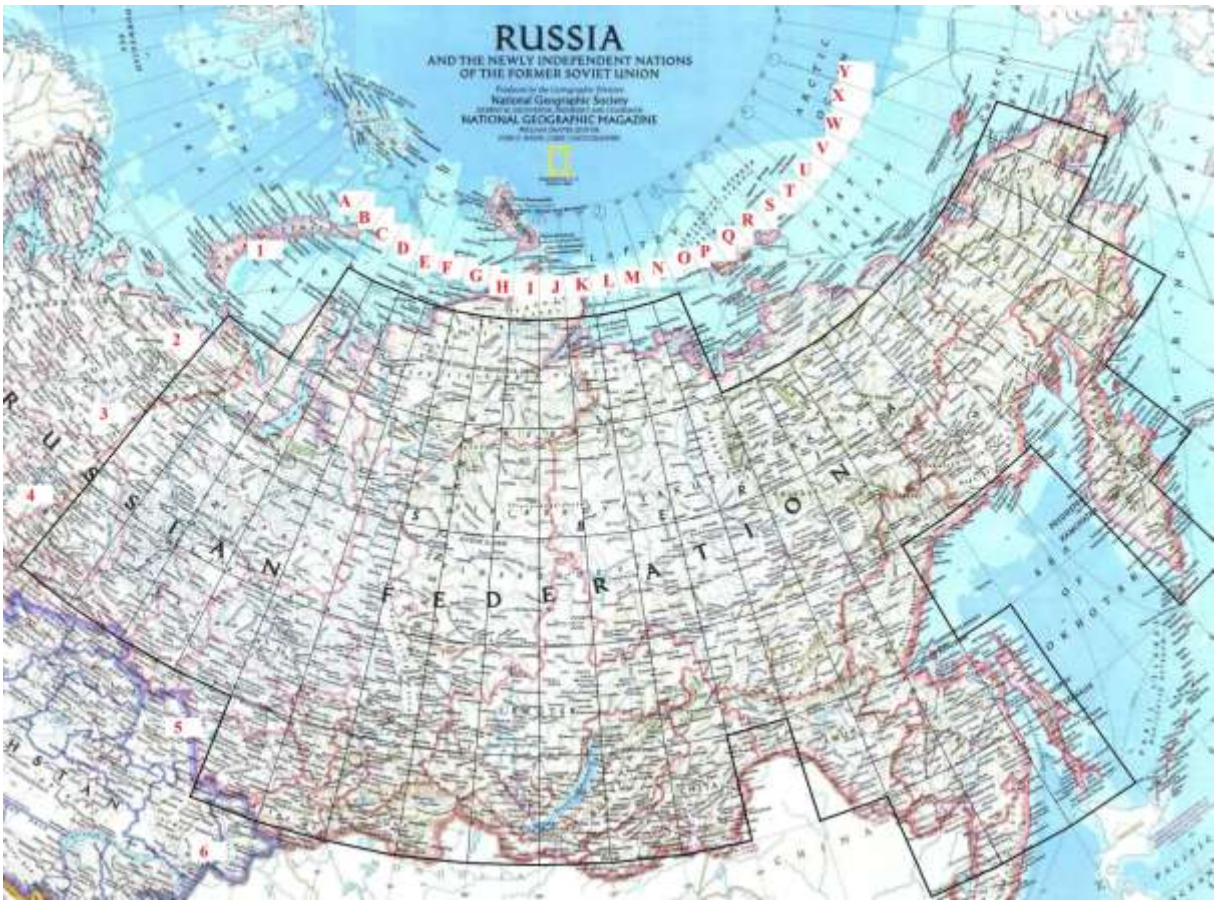
Our territory will be Siberia, which is extremely large (13 millions km2), with a very low density of population, which poses extra difficulties.

1. Geography

Siberia extends approximatively between 60° to 185° from West to East ; the territory is divided into 25 zones, designated by letters from A to Y ; each zone has a width of 5°.

Siberia extends between 50° to 75° from South to North ; the decomposition is made in 6 parallel strips, each representing 5°. The strips are numbered from 1 to 6, starting from the North.

Each elementary cell is therefore a "trapeze" of 5° x 5°. See the map below.



Important remark: the way the elementary cells are defined is certainly relevant for the resolution of the problem. Mathematically speaking, one might wish that the cells are as small as possible. But this is not possible in practice: one does not have the necessary information. So our choice is a compromise.

Each cell will be characterized by:

- The proportion of urbanized areas, denoted by U ;
- The proportion of cultivated areas: fields, meadows, various cultures, fruit trees, and so on, denoted by C ;
- The proportion of area which is still "virgin" : toundra, forests, and so on. It is denoted by V and its value is $V = 100 - U - C.$

Quite obviously, indeed, the probability of a fire is not the same, depending on the category of "occupation" of the soils. Data are given, for each cell, in the attached Excel file data_game_Siberia_2012_2013.xls.

Only the values of U and C are given, and the data are in percentage: 3 means 3% of the area of the cell. The value of V is $V = 100 - U - C$. Be careful: the area of a cell depends on its latitude!

We do not try to represent on a map the various types of soil occupation inside a cell: we only know the proportion for each of them.

2. Birth and propagation of a fire

For each of the three types of occupation (urban, cultivated, virgin), we know the probability of occurrence of a fire, per day. It is given by the following formulas:

- If the percentage of urban zones is $\geq 20\%$, the probability of occurrence of a fire is $p_1 = 2 \times 10^{-5}$ per km2 and per day;

- If the percentage of urban zones is $< 20\%$ and the percentage of cultivated zones is $\geq 20\%$, the probability is $p_2 = 0.5 \times 10^{-5}$ per km2 and per day;

- In all other cases, the probability is $p_3 = 10^{-6}$ per km2 and per day.

This concerns a given season, namely summer. More precisely, we fix a six month period, between April 1st till September 30th. We work only on this period. Quite normally, the repartition of the fight capacities may differ, according to the season.

The fire may occur at any point of the cell, with uniform probability on the cell. The fires appear independently from each other.

We admit that the detection of a fire is immediate, so is its localisation, which is precise (this is certainly not true in practice; there are extra difficulties connected with detection and localisation of fires and we do not take them into account here).

As soon as the fire occurs, it propagates itself. We admit (which is certainly oversimplified) that the propagation occurs everywhere with the same speed, which is 2 km per hour. The fire propagates as a circle, the center of which is the initial point.

The cost of damages depends on the burnt area. Per burnt square meter, we will take the following costs:

Urban Zone: 40 roubles
Cultivated Zone: 4 roubles
Virgin Zone: 0,4 roubles.

We admit that the zones which are burnt may burn again.

3. Fighting capacities

The objective of the game is to determine the repartition of the fighting capacities. They consist in ground Firemen and specially equipped planes.

– Ground Firemen

A Brigade consists in 30 men and 3 vehicles. Siberia has a total of 5 000 brigades (that is 150 000 men, 15 000 vehicles).

Each vehicle carries 6 000 liters of water, which is enough to extinguish 1 000 m2, after that it must get more water.

The average speed of the vehicles is 40 km/h. The diesel consumption is 30 l/100 km, and the price of a liter of diesel is 30 roubles. We admit that there are service stations every where.

Taking into account the fact that there are many lakes and rivers in Siberia, we admit that, no matter where the fire is, the distance in order to get a new supply of water is 100 km (in each direction). Water is free, does not require specific installations, and the delay for filling the tanks is not taken into account.

– Planes

In Siberia, there are 100 specialized planes for fires fight, of various types (IL 62, BE 200, helicopters, etc.)

We take the following average values:

– Each plane has a speed of 300 km/h ;
– Each plane carries 4 800 liters of kerosene;
– Each plane carries 6 000 liters of water, a quantity which may extinguish 1 000 m2 (0.1 ha); after that, it must go to a place to get more water.
– The flight autonomy is 2 400 km (consumption : 2 liters per km)

In order to get some kerosene, the plane must go to its ground basis, which is at the center of the cell, or to another center of a cell, if it is closer.

In order to get some water, the plane must travel for 100 km (each direction) ; indeed, as previously, there are numerous lakes and rivers.

The cost of kerosene is 30 roubles per liter, as before for gas-oil ; water is free.

4. Position of the fighting capacities

In each cell, the fighting capacities are positioned at the center of the cell, both for ground firemen and for planes.

5. Extinguishing a fire

We admit that the fire propagates as a circle with increasing radius. The rate of increase of this radius is 2 km/h, that is 0,56 m/s. But, as we explained above, the active zone (that is, the zone which is really burning) has only a width of 10 meters. Indeed, we consider that the center of the circled has already burned, only a corona is under fire. The area of the active zone, which firemen must extinguish, is therefore approximately:

$$S \simeq 2\pi\delta vt$$

where δ is the width of the active corona (10 m), v is the speed of propagation of the fire (0,56 m/s), t is the time elapsed since the beginning of the fire.

We admit that, in order to extinguish the fire, we have to cover the active zone (burning zone) with a height of water equal to $h = 0,6\,\text{cm}$. So, we need 6 liters per square meter; the amount of water needed in order to extinguish the fire is therefore, at time t:

$$Q(t) \simeq 2\pi\,\delta\,v\,h\,t$$

Extinguishing partly: If the firemen do not put enough water, a part of the corona is still under fire. We know the quantity of water used at time t, the size of the corona at that time, and so we can compute the area which is still under fire. We will consider that this area is that of a smaller corona, of same area. In other words, we consider that an insufficient quantity of water "shrinks" the fire, and brings it back to an earlier situation.

Pouring water: the water will always be poured at the central point, point where the fight started. The propagation has an importance with respect to the burned zone, but does not affect significantly the movements of the firemen. A change of 2 km is neglectible at the scale of Siberia. So, we admit the following facts:

- Both ground firemen and planes must always reach the starting point of the fire;
- This is the place where they put all water they have.
- If the quantity of water is insufficient, both ground firemen and planes must go and fetch more water, they come back to the starting point of the fire. The fire "contracted" due to earlier pouring of water, but it extended, when firemen left in order to get more water.

Capacities in one cell may be used in another cell. The distance must be computed on the Earth surface, which, at this scale, is not flat, but spherical. Radius of the Earth is approximately 6 370 km.

As we already said, we consider that a zone which already burned may burn again immediately.

6. The problem

The problem is to place the fight capacities (ground and planes) among the different cells, so as to minimize the expectation of total cost of burnings.

Once this optimal repartition is obtained, participants are asked to answer the following questions:

- Question 1 : What is the expectation of the total cost of burnings ?
- Question 2 : What is the expectation of the total budget for expenditures, that is gas-oil for vehicles and kerosene for planes ?
- Question 3 : What is the probability that the fight capacities prove to be insufficient ?

III. Comments

We tried to make our assumptions as precise as possible (and some of these assumptions are clearly over simplified, compared to real life situations). Still, the participants may find cases for which they will have to set complementary hypotheses. All hypotheses are acceptable, provided that they make sense and are explicitly set.

Participants should check from time to time the web sites of FFJM or SCM for possible precisions in the statements.

Many books, dealing with probabilities, have a presentation which is satisfactory in terms of axioms, but applications to real life situations are not clear, and sometimes contradictory. Our reference book here will be:

Bernard Beauzamy : Méthodes Probabilistes pour l'étude des phénomènes réels. Ouvrage édité par la Société de Calcul Mathématique SA, ISBN 2-9521458-0-6, ISSN 1767-1175 (in French).

IV. Participation rules

The game starts on November 1st, 2012 and ends on April 30th, 2013. Prizes will be given in May 2013, during the "Salon des Jeux Mathématiques", in Paris.

Participants should send their solution, in pdf format, in French or in English, no later than April 30th, 2013, to the email address: **ffjm@wanadoo.fr**

V. International Collaboration

This Competitive Game is a part of a collaborative project which SCM proposes to the Government of Novosibirsk Oblast, dealing with natural risks. This project received the support of the French Embassy in Moscow.