Société de Calcul Mathématique SA
Outils d'aide à la décision

Fédération Française des Jeux Mathématiques

Mathematical Competitive Game 2014-2015

Uncertainties in GPS Positioning

Fédération Française des Jeux Mathématiques
(French Federation of Mathematical Games)

and

Société de Calcul Mathématique SA

in partnership with

The French Institute for Transportation Science and Technology, Geolocalisation Team (IFSTTAR/CoSys/Geoloc)

and

The French Ministry of Transportation, Mission for Tarification Pricing (MEDDTL/DIGITIM/SAGS/MT)

Comments and results, by Bernard Beauzamy

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I. Participation

We received 26 individual participations and 7 group participations, representing a total of 54 people, plus a group of high school students who did not identify themselves.

The following countries were represented : Brazil, China, France, Germany, Great Britain, India, Russia, Slovenia, Spain, The Netherlands, USA.

So the participation to this 6th game was higher than before. We especially welcome high school students.

II. Presentation of the results

This is a real-life game, so the results should be presented in order to be understandable by an ordinary driver, using a GPS. This means that the position which was found should be put on a map, and not just given by coordinates. This position happens to be in the parking lot of IFSTTAR, west Brittany. The same way, the 90% set which is found should be described in some convenient way (either by a disk or, better in this case, by an ellipsoid) and drawn on a map.

Some participants just indicate equations in order to describe such a set. This is not convenient for practical use. If your GPS gave your position simply using coordinates, you would consider it as useless and send it back to the shop where you bought it, asking for a refund !

III. Mathematical approach

Most participants use a solver in order to find the approximate position of the receiver. This is indeed what a usual GPS does, but the usual GPS does not care about uncertainties, which are central to the present approach.

One should understand that the use of a solver (least square, for instance) is completely artificial. We do not know exactly where the receiver is, and there is no reason it might be exactly at the point which minimizes a sum of squares of errors. This is an arbitrary choice, which is not of probabilistic nature. The same, the solver indicates a time shift which is artificial : it might not be the true one.

It should be clear also that the solution to the first question (estimated position of the receiver) is not the best guess from the solver. The estimated position is, by definition, an expectation, computed as an average of all possible positions with their probabilities.

Many participants used a Monte Carlo method : throwing at random a large number of points, describing the various probability laws for the satellites and the various values for the pseudo-distances. After such choices have been made, they use the solver in order to find the position of the receiver, from these data. Such an approach is usually quite lengthy and, formally, it is not correct, because the position indicated by the solver is an arbitrary choice, with no probabilistic contents.

So, a true probabilistic approach requires no solver, no Monte Carlo method, and does not assume anything to be Gaussian.

A probabilistic approach goes as follows :

First, discretize the set where each satellite is: each satellite is in a sphere of radius 2 meters; discretize it using cubes of size 0.5 m. The probability of each cube is computed from the position of its center (or we could use the intersection with the sphere).

Second, find a rough set where the receiver must be, in the form of a parallelepiped (3d) or a rectangle (2d). Discretize it using cubes or squares of size 0.5 m (one may work in 3d or project upon a plane tangent to the Earth, if we decide to ignore the altitude). In order to do this, we observe that the intersection of spheres may be replaced by an intersection of planes (see link below).

Third, find a minimum and a maximum for the time shift, discretize the interval so that $c\tau \approx 0.5\,m$; a choice of a step of 10^{-9} s will result in a precision of 30 cm. Here, we have a uniform law on the time shift, because we know nothing about it a priori.

Then, the basic use of probabilistic methods goes as follows:

Take any position of the receiver (in step 2), any position of each satellite (in step 1), any value of the discretized time shift (in step 3). Compute, in that case, the geometric distance between the receiver and each satellite, and then the pseudo distance from the receiver to each satellite and associate to it a probability, which is the product:
proba (this value of the pseudo-distance) x proba (this value of the time shift) x proba (this position of satellite S1) x … x proba (this position of satellite S5).

For the position of the receiver chosen above, sum all these probabilities, for all values of the time shift and all positions of the satellites.

Do this for all positions of the receiver in the set defined in step 2 and renormalize (divide by the sum of all numbers, so the sum should be equal to 1). We now have a probability law for each position of the receiver. The expected position is the expectation of this probability law.

In order to find a 90% set, rank each position of the receiver by decreasing order of probability, compute the partial sums and stop when this partial sum is larger than 0.9.

A more detailed presentation can be found at :
http://www.scmsa.eu/archives/BB_GPS_Uncertainties_2014_04_19.pdf

IV. Results

1. Individuals

First Prize ex-aequo :

Christophe BIONDI, Marco TRUCCHI, from Nice, France
http://www.scmsa.eu/archives/Biondi_Trucchi.pdf

Alan OXLEY, United Kingdom
http://www.scmsa.eu/archives/OXLEYAlan.pdf

Second Prize
Helge DIETERT, University of Cambridge, Cambridge, United Kingdom

Third Prize
Michel Bénézit, Neuilly sur Seine, France

2. Groups

No first prize

Second Prize
Marco de Angelis, Hindolo George-Williams, Roberto Rocchetta
Graduate Students at the Institute for Risk and Uncertainty, University of Liverpool, United Kingdom

Third Prize
Sarah JEMMALI, Xintong SHAO, Meiyu XU, Anji ZHU, MAM4, Supervisor : Mr Julien BARRE, Polytech Nice-Sophia Antipolis, France.