Société de Calcul Mathématique SA
Outils d'aide à la décision

Fédération Française des Jeux Mathématiques

Mathematical Competitive Game 2013-2014

Checking an Industrial Process

Fédération Française des Jeux Mathématiques
(French Federation of Mathematical Games)

and

Société de Calcul Mathématique SA

Results and Comments by Bernard Beauzamy

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I. Generalities

The Game started on November 1st 2013 and finished on April 30th, 2014. The description was viewed by 22 697 people and 1 872 downloaded the data.

We received 12 Individual answers and 6 Group answers.

The following countries were represented in the answers :

Czech Republic, France, Great Britain, Portugal, South Africa, Spain, Switzerland, The Netherlands, Tunisia, USA.

II. Winners

The 2014 Prizes are as follows:

A. *Individuals*

1. First Prize : Michel Bénézit, Paris :
   to see the contribution, please click here

2. Second Prize : Miguel Nieves, United States Naval Academy

3. Third Prize : Nicola Mingotti, Spain

B. *Groups*

1. First Prize : S. Elaine Haley and Martin J. Mohlenkamp

Class MATH 5530, Statistical Computing Spring 2014; Department of Mathematics, Morton Hall 321, 1 Ohio University, Athens OH 45701 USA
   to see the contribution, please click here

2. No Second Prize.

3. Third Prize: The Miami Valley School Senior Multivariate Seminar, Centerville, Ohio, USA.

Christine Adib, Breanna Porter, Owen Robinette
Advisor: Ms. Betsy Witt
Consultants: Mr. Mark Ash, Mr. Lucas Witt

III. Technical comments

A. About the Game

The present Game was inspired by contracts we had with Air Liquide (industrial gases), ArcelorMittal (steel) and Areva (nuclear plants), though the data used here are completely fictitious.

Quite commonly, one cannot check all products coming out of a factory, because they are too numerous, or because it would be to slow or too costly to do so. An extreme case is of course when the testing process is destructive. Two situations may occur :

- The industry accepts some proportion of defective material, usually covered by a guaranty. This is the case of most common goods, such as cars, home appliances, and so on.

- The product is subject to severe safety regulations, which require, more or less clearly, that there is no defect at all. This is typically the case of the components in a nuclear plant.

The present Game deals with the second situation: the checking is hard to realize, very costly, and the Industry would like to make sure that there is no defect. Therefore, the definition of the sampling is essential. The question deals here with proportion of metals, but the techniques would be the same for any industrial concern (for instance, a minimum elastic deformation).

What the present Game shows is that a blind sampling (uniform law) is inefficient, even if it is commonly accepted by the Industry and the Safety Authorities.

A first remark is that, since the checking made by the Company is satisfactory, there is no use in performing usual statistical tests: they will show nothing. For instance, boostrap techniques, which only repeat the existing data, are useless.

A second remark is that fictitious assumptions are not allowed. One cannot make the assumption according to which the distribution of defects would follow a normal law, or a Poisson law, or whatever. We have data, and we have to interpret these data, with no "parametric assumption", as statisticians say.

So, we have to investigate the data.

We have here a product, which in an alloy, made of several metals. The question is to limit the variations in the composition. Quite clearly, these variations must depend on some physical laws, which may be complicated or even ignored. For sure, if the Industry knew quite well what imposes the variations, they would do their best to limit them. One can guess that temperature may play a role (so the beginning and the end of the production process, each day, are to be considered apart), and perhaps also gravity, which means that the bottom and top layers may differ ; contact with the walls of the recipient may also enter.

Here, we defined the following a priori laws (they are completely artificial):

$$Cr = 18 + \frac{1}{180}(1.1 + 2.2x)\exp(-y^2/2)(10 - z)\left(1 + \exp\left(\frac{-|t-1|}{1000}\right)\right)$$

$$Ni = 8 + \frac{1}{90}(1.1 + 2.2x)\exp(-y^2/2)(1 + z)\left(1 + \exp\left(\frac{-|t-1|}{1000}\right)\right)$$

where x, y, z are the coordinates of the center of the cell and t is the time in the day, starting at $t = 1$ each day.

The winners in each category correctly found these patterns.

We assume here that the measurements made both by the Industry and by the Safety Authorities are accurate: a discussion of these measurements is not required.

These laws indicate the variation of the composition with space and time. Even if the precise laws, described above, are difficult to reconstruct from the sampling (and would be impossible in real situations), still one sees some pattern from the sampling, and this pattern indicates where the improper cells might be. In our case, for instance, Cr increases with $x,$ decreases with $y, z, t.$ Therefore, the risk of high Cr is higher in the cells with high $x,$ low $y, z, t.$ One should do the same for the risk of low Cr, and the same for high and low $Ni.$

Even an approximate understanding of the pattern is sufficient to understand where the danger is and compute an approximate fine.

Here, the fine for completely incorrect cells is rather small, and the fine for too many "not too good" cells is close to zero. Still, there are cells which are defective. This shows that :

- The analysis process (that is, the sampling procedure) by the Industry is not appropriate;

- The control process by the Authorities is not satisfactory (amounts of cells to be checked, level of the fines).

B. Proper use of sampling by the Industry

Patterns, such as the one we saw, will appear from a first sampling made with a uniform law. At the beginning, we know nothing about the pattern, so we must explore all cells with equal probability: this is a uniform law. But once the pattern appears, then the next exploration should concentrate upon the cells which are regarded as "dangerous", and not with a uniform law anymore. And the third set of explorations should concentrate upon the cells which appeared after the first and second explorations, and so on.

Ideally, the $n-$th checking process, that is the designation of the cylinder to be checked at the $n-$th stage and the designation of the cells to be checked in it, should take into account the previous results. The first $n-1$ checkings result in a probability law, and the $n-$th stage is done according to this probability law. We used this approach in a contract with Air Liquide; when information is rare and difficult to obtain, every bit of it should be used in a dynamical process, so as to redefine the decision method at each step.

The blind checking process, used in the present Game, should be limited to situations where some proportion of rejections is acceptable. Such a process proves inadequate in situations dealing with safety concerns, for a reason which should be a posteriori obvious: these situations may be quite rare, may occur only in some "corners", and they will not be detected by random sampling, using uniform laws.

This, in particular, should cast strong doubts upon all safety checkings which are made using the so-called "Monte Carlo" methods.

Further reference:

[NMP] Bernard Beauzamy : Nouvelles Méthodes Probabilistes pour l'évaluation des risques. Ouvrage édité et commercialisé par la Société de Calcul Mathématique SA. ISBN 978-2-9521458-4-8. ISSN 1767-1175, avril 2010 (in French).