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REPLY TO
ATTENTION OF

Professor Bernard Beauzamy
Institut Calcul Mathématique
37 Rue Tournefort, 75005 Paris
France

October 25, 1994

Dear Professor Beauzamy,

I enclose with this letter a review of the paper entitled *Massively Parallel Computation of Many-variable Polynomial Interpolation*, written by Jérome Dégot as a portion of his doctoral dissertation from the Université de Lyon. As you can see from the comments, I believe that this work is mathematically sound and of a high quality. In addition, I believe that the problem of massively parallel polynomial interpolation is an important first step in the development of parallel algorithms for a number of problems (e.g., multivariable spline computations) which have important potential applications, including real time graphics and finite element analysis. If you need further written discussion from me regarding this work, please do not hesitate to contact me at your convenience.

I look forward to seeing you in Lyon in January for Dégot's defense and for further discussions on research in computational mathematics at ICM.

Sincerely,
Kenneth D. Clark

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Massively Parallel Computation of Many-variable Polynomial Interpolation

by
Jérome Dégot

I would like to first comment on the problem of polynomial interpolation, then make some specific comments regarding the aforementioned paper by Dégot.

Polynomial Interpolation

The problem of univariate interpolation has a long history, the results of which can be applied to many areas beyond the approximation of functions. Actually, it is a well known fact that arbitrary order polynomial interpolation can yield poor function representations for equally spaced nodes, so that unless the number of nodes is small or they are distributed according to some optimization criteria (such as the case of Chebychev), using polynomial interpolation for the purpose of function evaluation is ill-advised. In fact, in my experience the most important practical applications of polynomial interpolation lie not in the approximation of functions but rather in the intuition gained by studying interpolation for the study of splines and other bases of approximation as well as in the development of error estimates for numerical integration, collocation/finite element, and ODE solution techniques, with the consequent adaptive algorithms for these problems. Similar statements can be made for multivariate interpolation problems. Consequently, while a cleverly constructed massively parallel algorithm for polynomial interpolation may not have immediate practical application, I view this as an *important* first step in understanding the implications of parallelism on the construction of efficient algorithms for the manipulation of multivariate functions and data for numerous practical problems ranging from real time algorithms for computer graphics to finite element methods.

Dégot's Paper

This paper describes a massively parallel algorithm for the construction of a minimal degree multivariable interpolating polynomial at a prescribed set of N-dimensional complex data (nodes) for a set of n data. Based upon the hypercube representation of a polynomial and using Bombieri's scalar product of two polynomials and the corresponding induced Bombieri norm, the paper derives estimates of the sensitivity of the construction to perturbations of the nodes and/or data. This approach is very interesting since it yields important results of an analytic nature in a straightforward manner but also because of the reduction of computation of a polynomial to computation of its corresponding hypercube and the natural relation to parallel implementation. It should be noted, however, that there has been for some time considerable debate in the computational research community regarding the effectivity of various computing architectures, SIMD vs MIMD, shared memory vs. message passing, and so on. More recently, there is serious discussion and related research on heterogeneous distributed network-based computation (e.g., across a network of workstations possibly coupled to high performance mainframes such as a Cray or CM). Most likely, practical parallel algorithms resulting from Dégot's work will eventually be implemented in such a way and therefore must account for more open and flexible architectures. As a topic to be considered for future research, it would be of significant interest to build a graph theoretic interface between the hypercube representation of a polynomial and an abstract processor topology which could yield efficient parallel algorithms for polynomai computation for non-SIMD architectures. Finally, there has been some related research on the multivariate polynomial interpolation problem in the citations below (and references therein), although this work does not address the development of parallel algorithms, nor does it take the hypercube representation approach. These papers, and their descendants, deal mostly with approximation order, a separate issue which may suggest other future research directions.

In summary, this is a well written and mathematically sound paper, which has the potential to contribute significantly to development of efficient algorithms for polynomial computation "from the ground up" (i.e., bootstrapping) with parallelism as a motivating element.

Kenneth D. Clark
25 October 1994

References

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[2] deBoor, Carl and Amos Ron, *Computational aspects of polynomial interpolation in several variables*, preprint, 1990 (submitted to Math. Comp.).

[3] deBoor, Carl, *Multivariate piecewise polynomials*, Acta Numerica (1993), 65-109.