Société de Calcul Mathématique, S. A.
Algorithmes et Optimisation

Real Life Mathematics

Samples of exercises, coming from our experiences

June 2007

1. Bus stops

A bus line is 10 km long. The bus has an average speed of 20 km per hour between two stops. Let's assume that there is a bus stop every 200 m, so that everyone has a stop near his house. So everyone is happy. Let's say that stopping takes two minutes each time (time to slow down, to let people get out, get in, and to accelerate). How many stops do we have, and how long does it take to the bus to go from one end to the other ? Same question if the stops are every 500 m instead.

Comment : we see on this example that the general interest does not coincide with the sum of individual interests.

This comes from a work SCM made for Veolia Transport.

2. Pollution on a river

A river passes through 10 cities. Each city gives away some quantity of pollutant (for instance, phosphorus), and this quantity is measured by a station immediately after each city. How can you define the average pollution of the river ?

Comment : it cannot be the average of the indications given by each station, since each station shows all pollutions it receives.

This comes from a work SCM made for the European Environment Agency.

3. Effect of rainfall

Compare, from the point of view of effects, a rainfall of 5 mm per hour during one hour and a rainfall of 1 mm per hour during 5 hours.

This comes from a work SCM made for Veolia Environment, West Region, France.

4. Consumers' panel

A company sells sugar, each box weights 1 kg. For the year 2008, one million consumers are expected, but the number of boxes in unknown. The company asks a panel of 10 000 consumers. Among these 10 000,

- 20 % say that they will buy one box per month at most ;
- 40 % say from 2 to 5 ;
- 30 % say from 6 to 10 ;
- 10 % more than 10.

How can we predict the annual sales from this information ?

This comes from a work SCM made for Veolia Environment, West Region, France.

5. Uncertainties problems

A car moves with a speed of 60 km/h, so one km is done in one minute. But if we say that the distance is $1\text{ km } \pm 10\%$, that the speed is 60 km/h $\pm 10\%$ and that the time is one minute $\pm 10\%$, there is something strange. What ?

6. Line of sight

A ship moves in a uniform way : straight direction, constant speed. It sees a lighthouse at two different times, and records the angle for its direction. Taking into account the errors on this angle, we admit that the precision is $\pm 1^\circ$ each time. What position should be attributed to the lighthouse ? Same question if three or more experiments are made.

Comment : this problem is quite hard. The natural answer is : the intersection of the lines when we have only two. But this answer is wrong. Let us decompose each possible angle into ten small sectors (so we have eleven lines) ; the greenhouse may be on any of the intersections, and we have $11^2 = 121$ intersections, with equal probability. So the most likely position is the barycenter of these 121 positions, and this does not coincide with the intersection of the central lines of the sectors. See the book by Bernard Beauzamy "Méthodes probabilistes pour l'étude des phénomènes réels".

This comes from a work made by SCM for the "Service des Programmes de Missiles Tactiques, Délégation Générale pour l'Armement", Ministry of Defense, France.

7. Optimal trajectory for a robot

A robot cleans a swimming pool. It has caterpillars and a big circular brush, in the front part. We want the robot to pass twice at each place, in order to be sure that the bottom of the pool is correctly cleaned. Find a trajectory for the robot, in the case of a rectangular swimming pool.

This comes from SCM's work for Zodiac Pool Care, France.

8. Collecting garbage

An American-looking city is made of 20 streets, East-West and 30 avenues, North-South. Each block has exits in the four directions and disposes some garbage. Find the trajectory of a truck which must collect the garbage.

We can have a quantitative version of this problem : distances between streets, between avenues, unitary quantity of garbage, total volume of the truck. What should we do if one truck is not enough ?

Note : The optimal trajectory requires to pass once in each street and each avenue. This has nothing to do with the "traveling salesman" problems. A garbage collecting truck does not turn with acute angles.

9. Controlling a reservoir

A reservoir is alimented by a source, which has a periodic debit upon 24 hours (for instance, one liter per hour between 0 h and 8 h, 4 liters per hour between 8 h and 16 h, 2 liters per hour between 16 h and 24 h). We have a tap at the exit of this reservoir, and we can close or open this tap as we want. How should we act on this tap, so that the debit after the tap is as constant as possible, during the whole day ?

Indication : the constant value is obviously the average upon all entrance debits. There is a constraint upon the capacity of the reservoir.

This comes from a work made by SCM for the "Syndicat Interdépartemental d'Assainissement de l'Agglomération Parisienne" ; the debit on exit must be constant in order to allow the best possible treatment by the purification stations.

10. Managing production of electricity

We have at our disposal several power plants, for example 3. The first one may produce 800 MWh, per units of 100 MWh ; the second 900 per units of 300, and the third 1000 per units of 500. The production cost is 1 dollar for each MWh for the first, 1,2 for the second, 0,9 for the third. We need to produce 1934 MWh. What is the best repartition between the plants ?

Work realized by SCM for EdF (French Electricity).